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VERTICAL VELOCITY FIELD DUE TO LATENT HEAT OF CONDENSATION AND SENSIBLE HEAT FLUX — A CASE STUDY*

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ABSTRACT

For a few typical synoptic situations, vertical velocity is computed by quasi-geostrophic omega equation. The latent heat of condensation and sensible heat flux are parameterized in a way similar to the one used in operational quasi-geostrophic baroclinic model of Japan Meteorological Agency. Vertical velocities due to additional diabatic forcings are also computed separately.

It is found that the inclusion of latent heat of condensation further enhances the ascending adiabatic vertical velocities. The role of sensible heat flux, in quasi-geostrophic omega equation, does not seem to be of considerable importance unless the sea surface is warmer than the overlying air by about 10°C.

INTRODUCTION

VERTICAL velocity is one of the most valuable diagnostic tools to infer the weather and to understand the energetics of the atmospheric systems, and it is, therefore, but natural that several studies have been undertaken to compute the vertical velocity field, from the observed meteorological parameters. More emphasis, however, has been given on the dynamical methods compared to kinematic and adiabatic methods because of the basic advantage of the former due to their suitability of dynamic interpretation and understanding of the relative contribution of various forcings towards the total vertical velocity field.

The quasi-geostrophic-omega equation (Q.G.W.) is one of such dynamical diagnostic equations and has been widely used for computing the vertical velocity. In an earlier paper, author (Shukla, 1968) has given a detailed description of this equation and its method of numerical solution and therefore will not be discussed here in detail. It may however be remarked that there is sufficient evidence to suggest that the quasi-geostrophic omega equation may be applied even for the study of weather systems of low latitudes for which the Rossby number is less than 1 (Krishnamurty, 1968).

In order to study the relative contribution of all the forcings in the Q.G.W. equation in addition to the forcings due to parameterized latent heat of condensation and sensible heat flux, vertical velocities have been computed for several map times. The results will be presented here only for one map time i.e. 7 July 1963.

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Since the Indian subcontinent is surrounded on its three sides by water, it may be reasonable to believe that the sensible heat-flux from the warm oceanic waters may cause vertical velocities which may subsequently affect the weather over India. One of the primary objectives of the present study is to determine quantitatively the magnitude of the vertical velocities which may be caused due to the observed temperature differences between the sea-surface temperature and the overlying air temperature. The computed vertical velocities indicate that for the observed temperature differences at the air-sea interface, over the Indian Ocean which is about 1·3°C, the contribution towards the vertical velocity, as obtained through diagnostic omega equation is not appreciable. The contribution of the latent heat of condensation is, however, as significant as the combined contribution of the adiabatic forcings.

TABLE 1. *List of symbols*

C_p	Coefficient of specific heat at constant pressure	T_a	Air temperature
f	Coriolis parameter	T_s	Sea-surface temperature
L	Latent heat of condensation	η	Absolute vorticity
p	atmospheric pressure	ω	The individual change of pressure
R	Specific gas constant	ϕ	geopotential
S	Static Stability	∇^2	Laplacian operator
T	Temperature	J	Jacobian operator

THE OMEGA EQUATION

Following the notations given in Table 1, the quasi-geostrophic omega equations for adiabatic motion may be written as

$$\nabla^2 \omega + \frac{f_0^2}{S} \frac{\partial^2 \omega}{\partial p^2} = \frac{1}{S} \left| \frac{\partial}{\partial p} J(\phi, \eta) + \frac{1}{f_0} \nabla^2 \left\{ J(\phi, -\frac{\partial \phi}{\partial p}) \right\} \right| \ldots \quad (2.1)$$

The omega equation is found by combining the vorticity and thermodynamic energy equation under the usual quasi-geostrophic approximations based on the scale-analysis given by Charney (1948). The first term on the R.H.S. is the differential vorticity advection and the second term is the laplacian of thermal advection. Since the above equation is linear in ω, two separate solutions can be found corresponding to both of these terms separately.

For deriving the above equation, non-adiabatic heating term which appears in the thermodynamic energy equation has been assumed to be zero. However, if the heating term is also included in the thermodynamic energy equation, the resulting omega equation is given as

$$\nabla^2 \omega + \frac{f_0^2}{S} \frac{\partial^2 \omega}{\partial p^2} = \frac{1}{S} \left| \frac{\partial}{\partial p} J(\phi, \eta) + \frac{1}{f_0} \nabla^2 \left\{ J(\phi, -\frac{\partial \phi}{\partial p}) \right\} - \frac{R}{C_p . p} . \nabla^2 \frac{dQ}{dt} \right| \ldots (2.2)$$

where $\frac{dQ}{dt}$ is the rate of heating per unit time and unit mass. $\frac{dQ}{dt}$ may be further broken into two components due to latent heat of condensation and sensible heat flux.

$$\frac{dQ}{dt} = \frac{dQ}{dt} + \frac{dQS}{dt} \qquad \ldots \quad (2.3)$$

where $\frac{dQL}{dt}$ and $\frac{dQS}{dt}$ are the rate of heating per unit time and unit mass due to latent heat of condensation and sensible heat flux respectively.

Parameterization of sensible heat flux :

The parameterization used here is similar to the one used in the operational 4 level quasi-geostrophic boraclinic model used in Japan (1963). In this parameterization, $\frac{dQS}{dt}$ is given as :

$$\frac{dQS}{dt} = A \mid V \mid (T - T_a)\left(\frac{p}{p^*}\right)^v \qquad \ldots \quad (2.4)$$

V is wind speed, P^* is surface pressure and A, ν are constants. A has been taken to be

$$A = 10^{-3} \text{ m sec}^{-1} \text{ deg}^{-1} \quad \text{if } T_s > T_a$$
$$\text{and } A = 10^{-4} \text{ m sec}^{-1} \text{ deg}^{-1} \quad \text{if } T_s < T_a$$

Parameterization of latent heat of condensation :

Following the same method of parameterizing the latent heat of condensation as used in the operational 4-level quasi-geostrophic baroclinic model in Japan (1963), $\frac{dQL}{dt}$ may be given as

$$\frac{dQL}{dt} = - L \frac{dq^*}{dt}$$

where L is the latent heat of condensation and q^* is the saturated specific humidity. Following Gambo (1963), the above equation may be written as

$$\frac{dQL}{dt} = - \omega L F^* \qquad \ldots (2.5)$$

where

$$F^* = \frac{1}{1 + \frac{L}{Cp}\left(\frac{\delta q^*}{\delta T}\right)_p} \left| \left(\frac{\delta q^*}{\delta p}\right)_T + \frac{XT}{p}\left(\frac{\delta q^*}{\delta T}\right)_p \right| (X = R/Cp)$$

In the above expression, subscripts p and T denote the differentiation at p=constant and T=constant respectively. It may be seen that the R.H.S. of equation (2.5) contains ω which is yet to be determined. For computing the vertical velocity due to forcing of latent heat of condensation, the value of ω obtained by solving (2.1) is used in the R.H.S. of (2.5).

Numerical Solution of Omega Equation

The numerical solution of the following equations will give the vertical velocity due to the four forcings on the R.H.S.

$$\nabla^2 \omega_1 + \frac{f_0^2}{S} \frac{\partial^2 \omega_1}{\partial p^2} = \frac{1}{S} \left\{ \frac{\partial}{\partial p} J(\phi, \eta) \right\} \quad \ldots \quad (3.1)$$

$$\nabla^2 \omega_2 + \frac{f_0^2}{S} \frac{\partial^2 \omega_2}{\partial p^2} = \frac{1}{f_0 S} \nabla^2 \left\{ J\left(\phi, -\frac{\partial \phi}{\partial p}\right) \right\} \quad \ldots \quad (3.2)$$

$$\nabla^2 \omega_3 + \frac{f_0^2}{S} \frac{\partial^2 \omega_3}{\partial p^2} = -\frac{1}{S} \frac{R}{Cp.p} \nabla^2 \left(\frac{dQs}{dt} \right) \quad \ldots \quad (3.3)$$

$$\nabla^2 \omega_4 + \frac{f_0^2}{S} \frac{\partial^2 \omega_4}{\partial p^2} = -\frac{1}{S} \frac{R}{Cp.p} \nabla^2 \left(\frac{dQL}{dt} \right)$$

$$= \frac{1}{S} \frac{R}{Cp.p} \nabla^2 (\omega^* LF^*) \quad \ldots \quad (3.4)$$

where $\omega^* = \omega_1 + \omega_2 + \omega_3$.

In the above equation, ω_1, ω_2, ω_3 and ω_4 are the vertical velocities due to the forcings of differential vertical advection, laplacian of thermal advection, sensible heat flux and latent heat of condensation respectively.

The above equations have been solved by the method of three dimensional relaxation using an overrelaxation coefficient of 0.3 and tolerance for the residual to be 0.00001 mb/sec. For a detailed description of the finite difference form and numerical solution of the above equations, one may refer to an earlier paper by the author (Shukla, 1968).

Data, grid length and vertical resolution of the model:

The results of computations which have been presented in this article are for 00 Z of 7 July 1963. Geopotential field is specified at the levels 900 mb, 700 mb, 500 mb and 300 mb. Boundary conditions for the omega are specified at 1000 mb for the lower boundary and at 200 mb for the upper boundary. The vertical velocities are computed at the levels 800 mb, 600 mb, and 400 mb. Geopotential values are picked up from the manually analysed charts at 2.5 degree's latitude—longitude intersections for the area between 2.5°N through 40°N and 50°E through 100°E. Fig. 1 (a) and (b) give the input geopotential field for 700 mb and 500 mb respectively. To save the space, only a few charts are being presented in the paper.

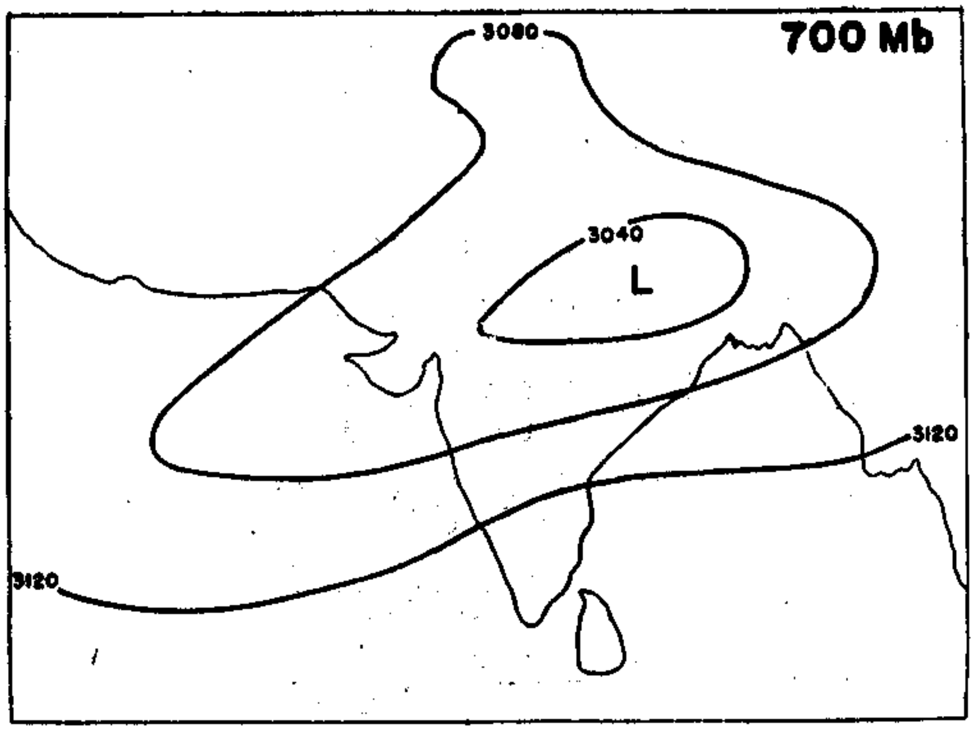

Fig. 1 a. Geopotential (ø') Field at 700 mb on 7.7.63 (ooz).

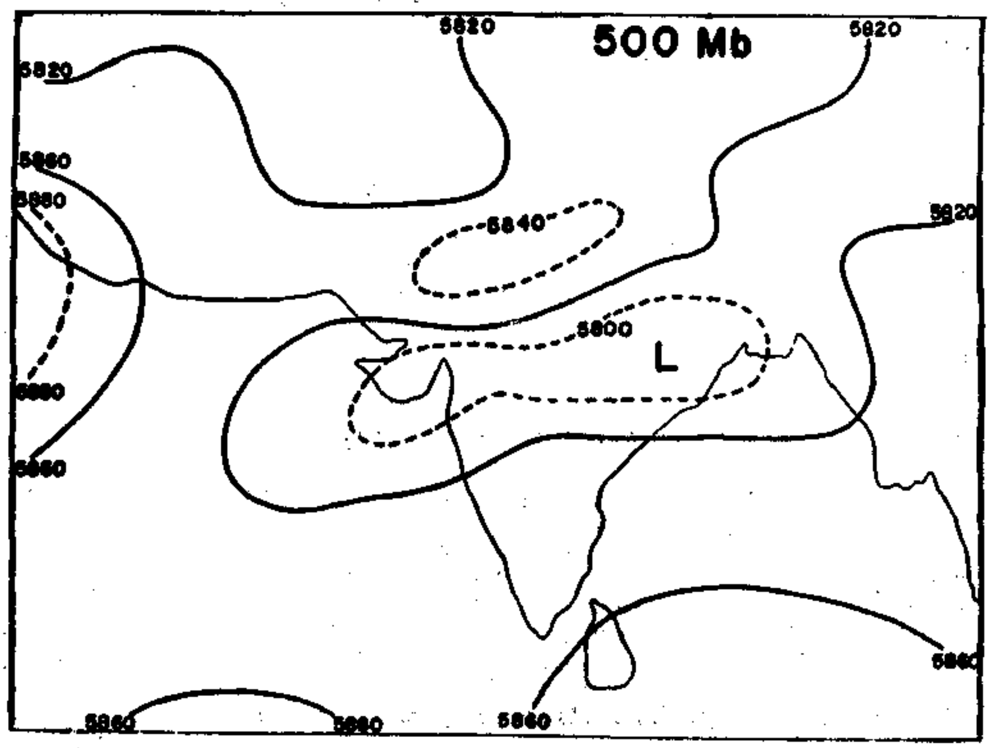

Fig. 1 b. Geopotential (ø') Field at 500 mb on 7.7.63 (ooz).

Boundary conditions :

ω has been taken to be zero at all the lateral boundaries and also at the uppermost boundary. The vertical velocity at the lower boundary, ω_b has been taken to be

$$\omega_b = \vec{V} \cdot \nabla P_s$$

where $\vec{V}$ is the surface wind vector and P_s is the surface pressure. It may be incidentally remarked that this method of specifying the lower boundary condition does not appear to be appropriate for Western Ghats because the grid length itself is larger than the width of the mountains and therefore it may be desirable to use a finer grid (say 100 km) for evaluating the terrain induced vertical velocity. The method also does not seem to be appropriate for Himalayan mountain because of its large size as it may not be realistic to assume that all the air, being obstructed from the mountain, goes up. It may, therefore, be desirable to specify more realistic boundary conditions which take into account also the motion of the air which may go around the mountain.

Results of Vertical Velocity Computations

As mentioned earlier, vertical velocity has been calculated at 800 mb, 600 mb and 400 mb. However, in order to save the space, the vertical velocity for the middle level i.e. 600 mb only will be presented here. Fig. 2 gives the vertical velocity due to the adiabatic forcings only. In other words, Fig. 2 gives the algebraic sum of

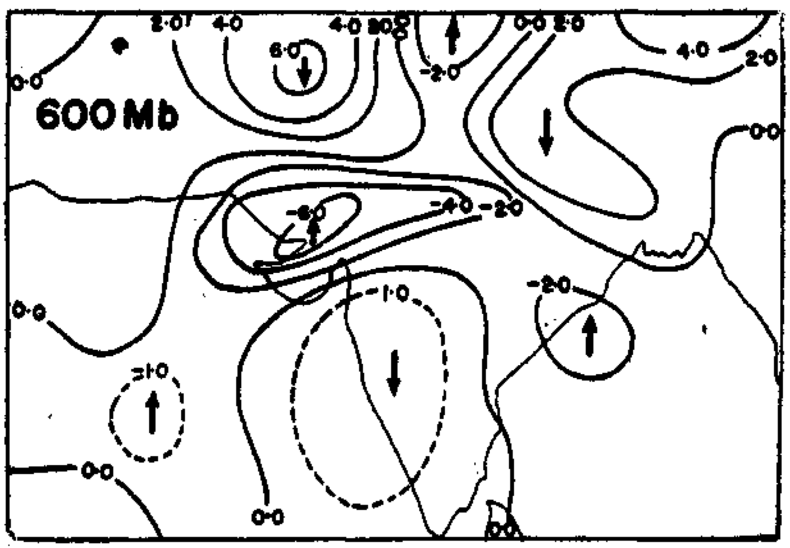

Fig. 2. Vertical velocity field at 600 mb in the units of mb/Hour, due to adiabatic forcings (Nagative values mean rising motion).

ω_1 and ω_2. Fig. 3 gives the vertical velocity due to the forcings of latent heat of condensation. Fig. 4 gives the vertical velocity due to combined effects of adiabatic forcings and latent heat of condensation.

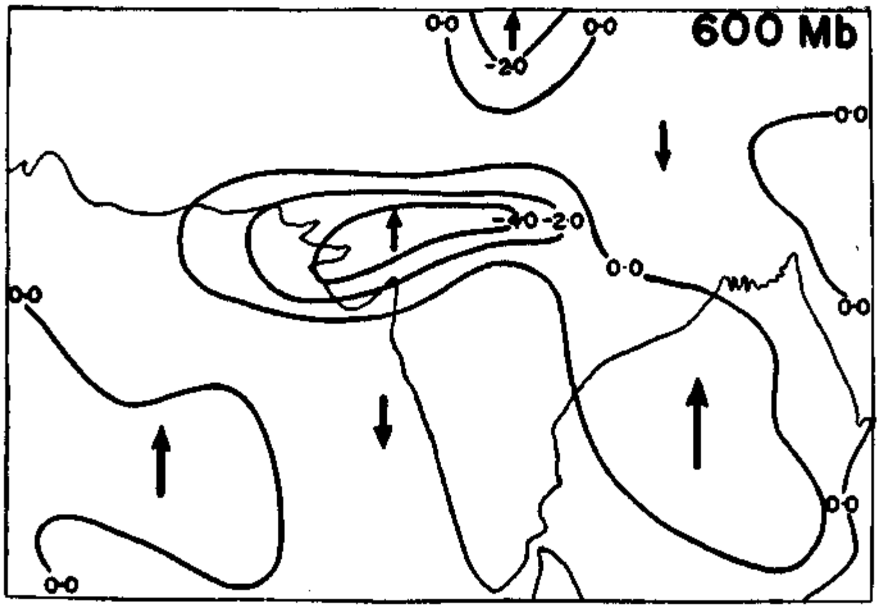

Fig. 3. Vertical velocity field at 600 mb in the units of mb/Hour, due to forcing of latent heat of condensation (Negative values mean rising motion).

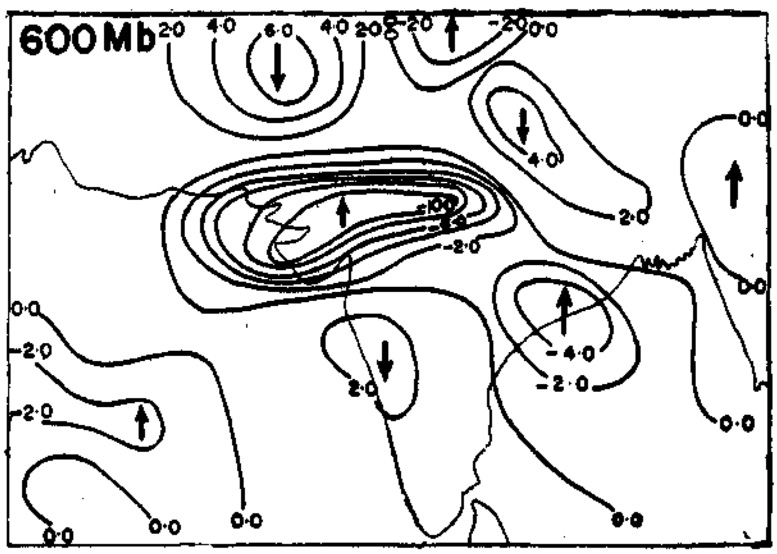

Fig. 4. Vertical velocity field at 600 mb in the units of mb/Hour, due to combined effect of adiabatic and diabatic forcings (Negative values mean rising motion).

It may be noted that the vertical velocity due to the forcing of sensible heat flux have not been presented in a separate diagram because the vertical velocities, which were obtained corresponding to the observed temperature differences between the sea surface and the overlying air, were found to be even less than 0.1 mb hr^{-1}.

Discussion

The patterns and magnitudes of the vertical velocity field obtained corresponding to various individual forcings at different levels present a lot of interesting material of which a very detailed discussion may be presented. However, to make the presentation brief, only salient points will be presented here.

Table 2 gives the actual values of vertical velocities in the units of mb/hour at 7 grid points along the meridian 72.5°E at 600 mb corresponding to the various forcing functions. It is seen that the forcing of the laplacian of thickness advection is the most dominant. The differential vorticity advection does not contribute much, probably because the vorticity values are almost same at different levels in the vertical. It is also noticed that the contributions of latent heat of condensation is comparable to that of adiabatic vertical motion. In this parameterization, the ascending motions are enhanced by the inclusion of latent heat of condensation. The descending motions are not affected. This parameterization has some limitations in respect of not considering the small-scale convection which primarily is responsible for the latent heat release. It is, however, believed that although the monsoon rainfall is mainly due to broad-scale convective activity, this activity takes place only in such preferred zones in which large scale vertical velocity exists. The solution of omega equation, being incapable of resolving small scale convection, only delineates such areas of large scale vertical velocity, in which, small scale convection may be embedded. In this parameterization it is also assumed that condensation takes place at all the grid points where adiabatic omega equation gives rising motion. Since the fluctuations in the moisture field have been noticed to be quite large even in the peak-monsoon months, this assumption may not be quite valid. This deficiency may, however, be rectified by putting an additional constrain on the condensation with respect to the percentage of relative humidity so that condensation is allowed only at those grid points where in addition rising motion, the relative humidity is equal to or greater than 80 per cent.

The contribution of the forcing of sensible heat flux is rather negligible. This is probably because of the observed smaller differences between the sea surface and the overlying air temperatures.

A comparison of Fig. 4 with the observed rainfall on 7th July, revealed that the areas of widespread rainfall coincided with the regions of rising motion and the areas where no rainfall was recorded were the regions of sinking motion. The only exception to this was the west coast,rainfall. This was because the west coast rainfall is mostly orographic and because of inadequate resolution in the present model, this factor has not been accounted well. It is proposed to devise some empirical formulation which, when used in conjunction with the omega equation, will give the adequate vertical motions even at the western ghats. Similar approach will be needed for the orographic rainfall of Arakan Coast also.

TABLE 2. *Contribution to the vertical motion by different forcing functions*

	Forcing Function	Description of the forcing function	Vertical motion of 600 mb at points along 72.5°E in unit of mb/hour						
			15°N	17.5°N	20°N	22.5°N	25°N	27.5°N	30°N
1	$\frac{1}{S}\left\{\frac{\partial}{\partial p} J(\phi, \eta)\right\}$	differential vorticity advection.	—0.1	—0.5	—1.1	—1.6	—0.3	0.3	0.2
2	$\frac{1}{f_0 S} \nabla^2 \left\{J\left(\phi, -\frac{\partial \phi}{\partial p}\right)\right\}$	Laplacian of thermal advection.	1.7	3.2	3.2	—2.1	—5.7	1.0	3.4
3	$-\frac{1}{S}\frac{R}{Cp.p} \nabla^2 \left(\frac{dQ_s}{dt}\right)$	Diabatic heating (Sensible heat)	0	0	0	0	0	0	0
4	$-\frac{1}{S}\frac{R}{Cp.p} \nabla^2 \left(\frac{dQL}{dt}\right)$	Diabatic heating (latent heat)	0.4	0.8	1.2	—2.6	—4.7	1.7	1.2
5	(1. + 2. + 3. + 4.)	All forcings combined	2.0	3.5	3.3	—6.3	—10.7	3.0	4.8

(Negative value of omega means rising motion and positive value of omega means sinking motion).

CONCLUDING REMARKS

The present study indicates the feasibility of using quasi-geostrophic omega equation for diagnostic studies in order to delineate the regions of large scale rising and sinking motion. Such results may be of great importance even in routine forecasting because it is in these regions of large scale rising motion that intense convective activity takes place and causes the monsoon rainfall. It is, however, suggested that further improvements must be made in the parameterization of diabatic heating, orographic and frictional effects. It would also be interesting to compute the vertical velocity by balance-baroclinic-omega-equation and compare it with the quasi-geostrophic-omega-equation.

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